Weakly bound systems, continuum effects

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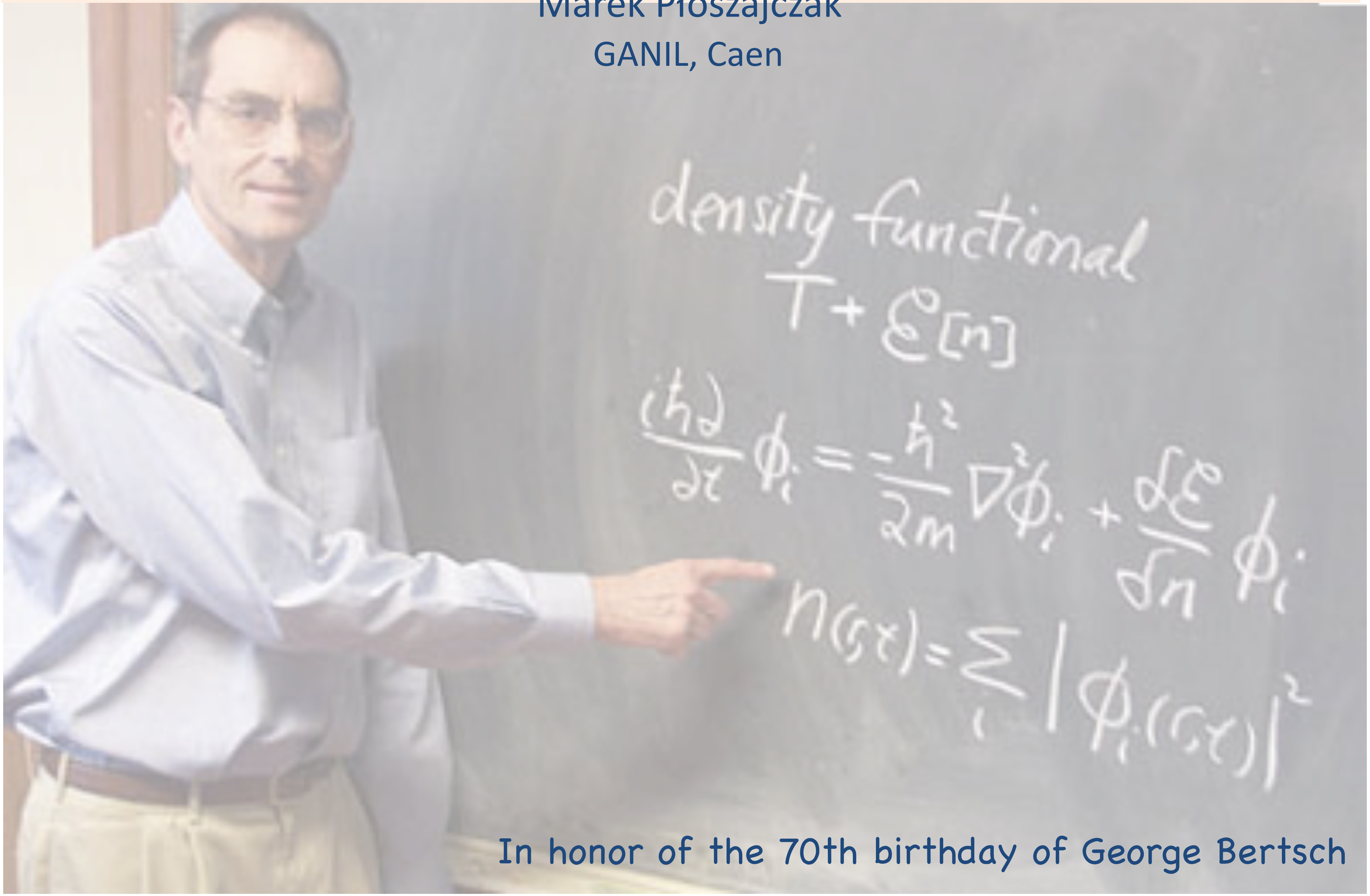
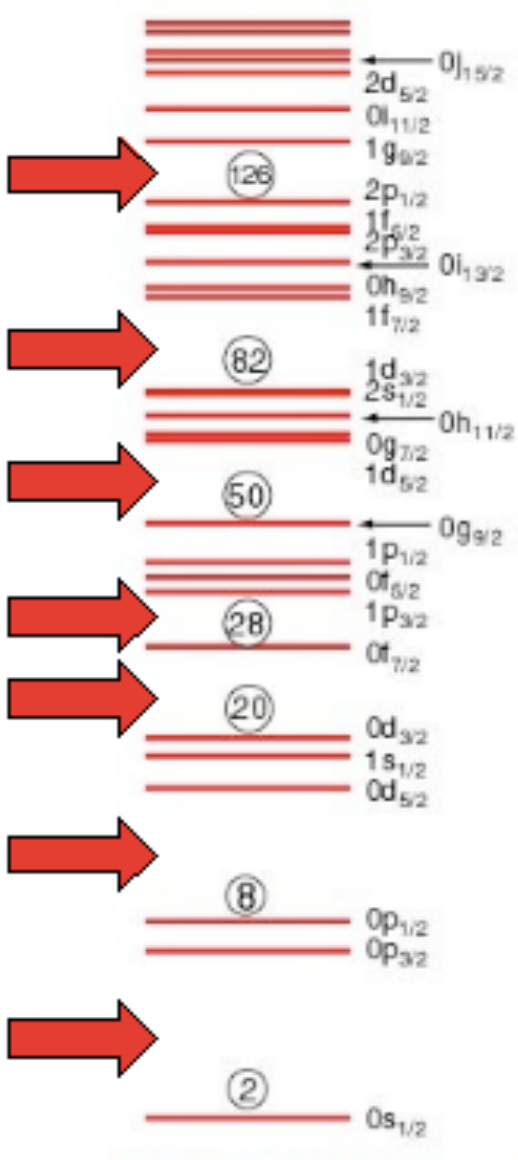


In honor of the 70th birthday of George Bertsch

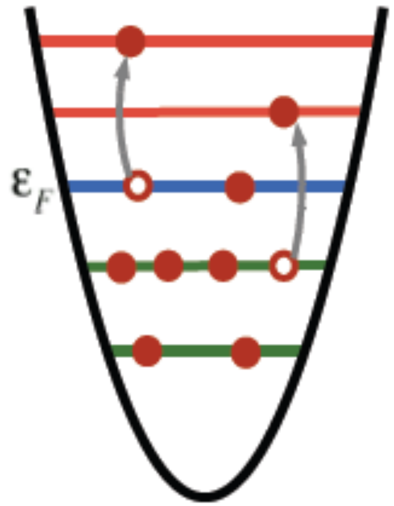

FUSTIPEN
French-U.S. Theory Institute for Physics with Exotic Nuclei



Shell Model of Nuclei

How it all begun…?

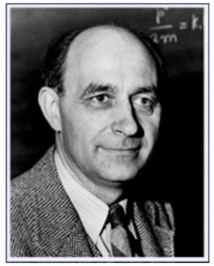
Closed Quantum System
No coupling with decay channels



To what extent the change in boundary conditions at the nuclear surface due to Coulomb wave function distortion in the external region can explain relative displacement of states in mirror nuclei?

J.B. Ehrman (1950)

Role of boundary conditions in universal properties of reaction cross-sections at the threshold E.P. Wigner (1948)



Enrico Fermi

Maria Goeppert-Mayer

J. Hans D. Jensen

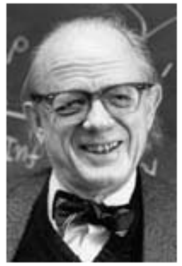
Eugene P. Wigner

ANNALS OF PHYSICS: 19, 287–313 (1962)

A Unified Theory of Nuclear Reactions. II*

HERMAN FESHBACH

Department of Physics and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts



The principal device employed, as in part I, is the projection operator which selects the open channel components of the wave function...

A unified approach to nuclear structure and reactions

C. Mahaux, H.A. Weidenmüller, *Shell Model Approach to Nuclear Reactions (1969)*
H.W.Bartz et al, Nucl. Phys. A275 (1977) 111
K. Bennaceur et al, Nucl. Phys. A651 (1999) 289
J. Rotureau et al, Nucl. Phys. A767 (2006) 13
…

Open QS solution in Q :

$$\mathcal{H}^{\text{eff}}_{\text{QQ}}|\Psi_\alpha\rangle = \mathcal{E}_\alpha(\text{E},\text{V}_0)|\Psi_\alpha\rangle$$
$$\langle\Psi_{\tilde\alpha}|\mathcal{H}^{\text{eff}}_{\text{QQ}} = \mathcal{E}^*_\alpha(\text{E},\text{V}_0)\langle\Psi_{\tilde\alpha}|$$

$$\langle\Psi_{\tilde\alpha}|\Psi_\beta\rangle = \delta_{\alpha\beta}$$

$$\mathcal{H}^{eff}_{\mathcal{QQ}}(\text{E}) = \text{H}_{\mathcal{QQ}} + \text{H}_{\mathcal{QP}}\frac{1}{\text{E}-\text{H}_{\mathcal{PP}}}\text{H}_{\mathcal{PQ}}$$

$$\Psi_\alpha = \sum_i b_{\alpha i}\Phi_i^{(\text{SM})}$$
$$\boxed{\Psi_E^c \sim \sum_\alpha c_\alpha \Psi_\alpha}$$

For bound states: $\mathcal{E}_\alpha(E)$ is real and $\mathcal{E}_\alpha(E) = E$
For unbound states: physical resonances $\equiv$ poles of S-matrix

PHYSICAL REVIEW VOLUME 124, NUMBER 6 DECEMBER 15, 1961

Effects of Configuration Interaction on Intensities and Phase Shifts*

U. FANO
National Bureau of Standards, Washington, D. C.
(Received July 14, 1961)

The actual stationary states may be represented as superpositions of states of different configurations which are "mixed" by the "configuration interaction," i.e., by terms of the Hamiltonian that are disregarded in the independent-particle approximation. The effects of configuration interaction are particularly conspicuous at energy levels above the lowest ionization threshold, where states of different configurations coincide in energy exactly since at least some of them belong to a continuous spectrum. The mixing of a configuration belonging to a discrete spectrum with continuous spectrum configurations gives rise to the phenomenon of *autoionization*. The exact coincidence of the energies of different configurations makes the ordinary perturbation theory inadequate, so that special procedures are required for the treatment of autoionization and of related phenomena.

U. Fano

The Hilbert space includes bound and scattering states → Resonance spectrum is discarded as unphysical

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U. Fano

The achievement of this goal took ~40 years and required the development of:

I.M. Gelfand

- New mathematical concepts: Rigged Hilbert Space (≥1964),...

T. Berggren

- Generalized completeness relation including s.p. bound states, resonances, and scattering states (~1968)

- New many-body framework(s): Gamow Shell Model (2002),...

$$i\hbar\frac{\partial}{\partial t}\Phi(r,t)=\hat{H}\Phi(r,t)\ ;\quad \Phi(r,t)=\tau(t)\Psi(r)$$

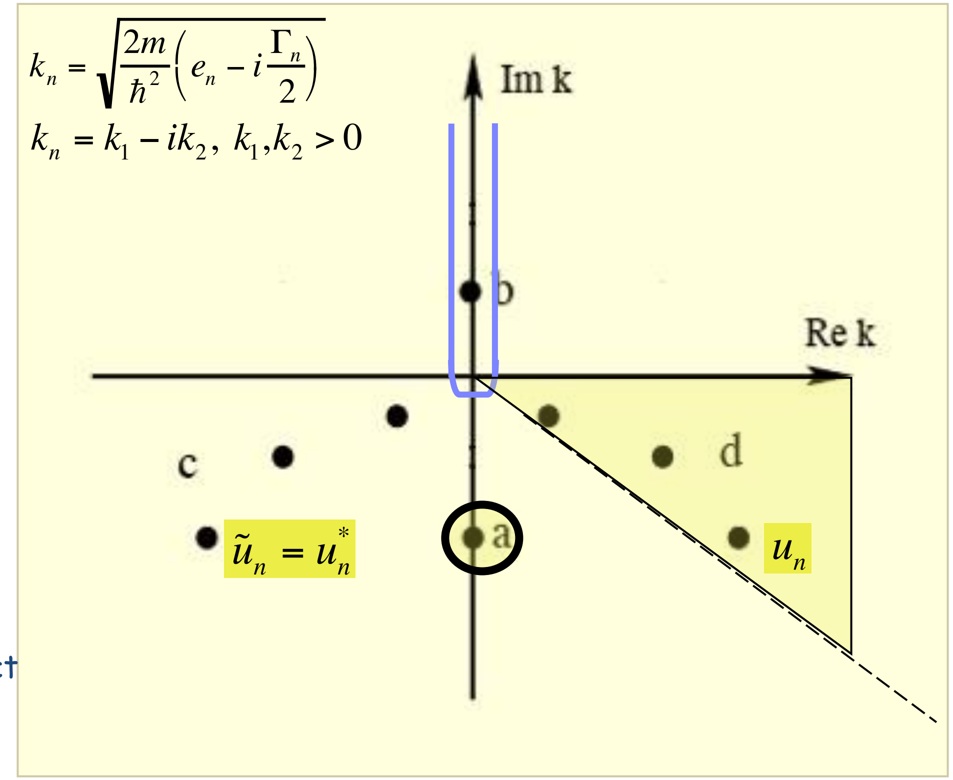
$$\hat{H}\Psi=\left(e-i\frac{\Gamma}{2}\right)\Psi \quad \rightarrow \quad \tau(t)=\exp\left(-i\left(e-i\frac{\Gamma}{2}\right)\right)$$

$$\Psi(0,k)=0\ ,\quad \begin{cases}\Psi(\vec{r},k)\underset{r\to\infty}{\rightarrow}O_l(kr)\\ \Psi(\vec{r},k)\underset{r\to\infty}{\rightarrow}I_l(kr)+O_l(kr)\end{cases}$$

Only bound states are integrable!

Euclidean inner product $\qquad$ Rigged Hilbert Space inner product

$$\langle u_n|u_n\rangle=\int_0^\infty dr\,u_n^*(r)u_n(r) \longrightarrow \langle\tilde{u}_n|u_n\rangle=\int_0^\infty dr\,\tilde{u}_n^*(r)u_n(r)$$



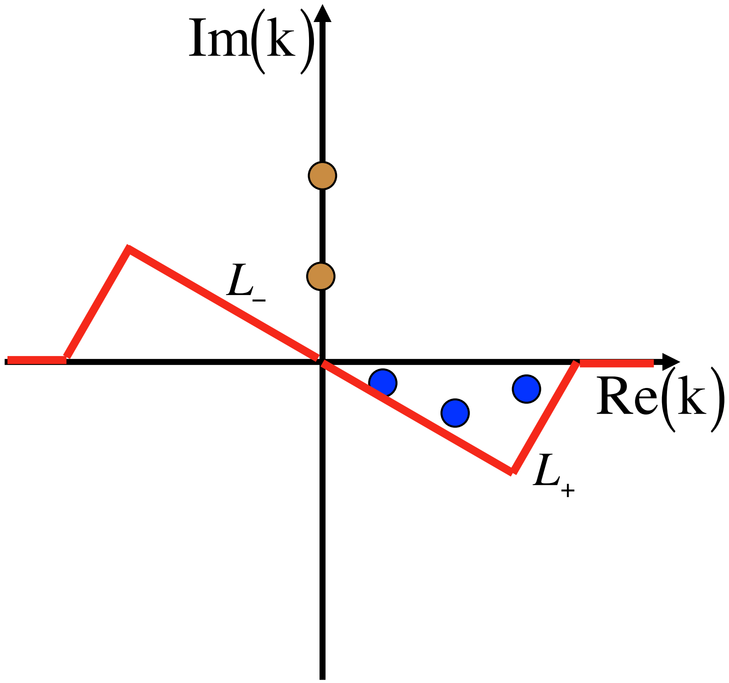
Rigged Hilbert Space (RHS) is the natural setting of Quantum Mechanics in which resonance spectrum, Dirac bra-ket formalism (and Heisenberg uncertainty relations) have place

I.M. Gel'fand and N. J. Vilenkin. *Generalized Functions*, vol. 4: *Some Applications of Harmonic Analysis. Rigged Hilbert Spaces*, Academic Press, New York, 1964

G. Ludwig, *Foundation of Quantum Mechanics*, Vol. I and II, Springer-Verlag, New York, 1983

Completeness relation

T. Berggren, Nucl. Phys. A109, 265 (1968)



$$\sum_n |u_n\rangle\langle\tilde{u}_n| + \int_{L_+} |u_k\rangle\langle\tilde{u}_k|\,dk = 1 \quad ; \langle u_i|\tilde{u}_j\rangle = \delta_{ij}$$

bound states
resonances

non-resonant
continuum

$$|SD_i\rangle = |u_{i_1} \ldots u_{i_A}\rangle \rightarrow \sum_k |SD_k\rangle\langle\widetilde{SD}_k| \cong 1$$

$$H \rightarrow [H]_{ij} = [H]_{ji}$$

complex-symmetric eigenvalue problem for hermitian Hamiltonian

Gamow Shell Model

N. Michel et al, PRL 89 (2002) 042502; PRC 79, 014304 (2009)
R. Id Betan et al, PRL 89 (2002) 042501

+

Density Matrix Renormalization Group method

J. Rotureau et al., PRL 97, 110603 (2007); PRC 79, 014304 (2009)

Other applications:

Continuum Coupled Cluster approach

G. Hagen et al, PLB 656, 169 (2007)

No-Core Gamow Shell Model

G. Papadimitriou, J. Rotureau et al (2012)

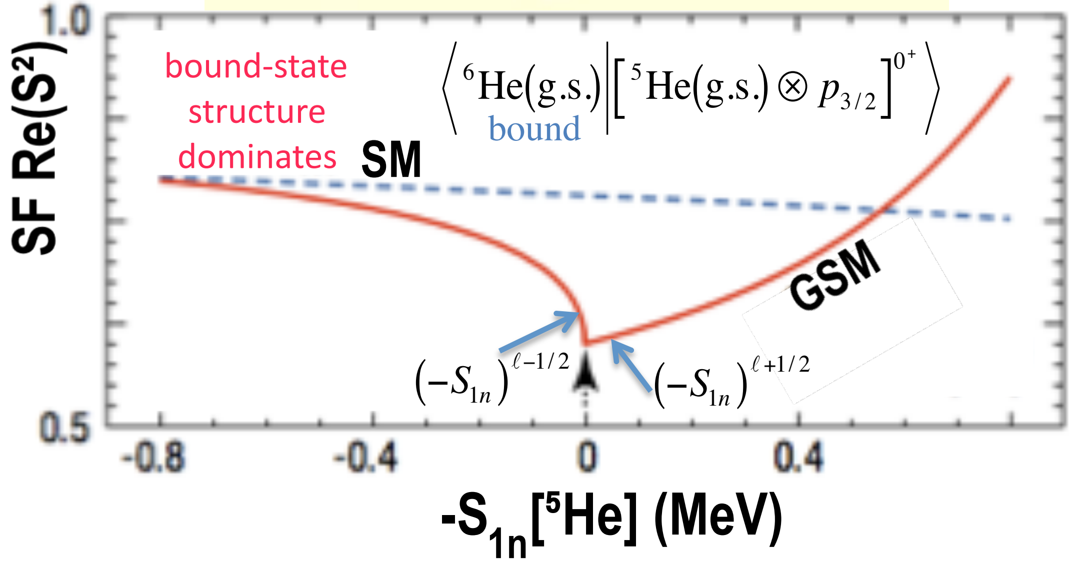
The interplay between Hermitian and anti-Hermitian couplings is a source of collective effects

- resonance trapping — P. Kleinwächter, I. Rotter, PRC 32, 1742 (1985)
- super-radiance phenomenon — N. Auerbach, V.G. Zelevinsky, Rep. Prog. Phys. 74, 106301 (2011)
- modification of spectral fluctuations — Y.V. Fyodorov, B.A. Khoruzhenko, PRL 83, 65 (1999)
- multichannel coupling effects in reaction cross-sections and shell occupancies

N. Michel, W. Nazarewicz, M.P., PRC 75, 031301 (2007)

Example:

$$S^2 \equiv \int u_{\ell j}^2(r)dr = \sum_{\mathcal{B}} \langle \widetilde{\Psi_A^{J_A}} || a_{\ell j}^{+}(\mathcal{B}) || \Psi_{A-1}^{J_{A-1}} \rangle^2$$



Analogy with the Wigner threshold phenomenon for reaction cross-sections

E.P. Wigner, PR 73, 1002 (1948)

$$Y(b,a)X : \sigma_\ell \sim k^{2\ell-1} \quad \Longleftrightarrow \quad (-S_n)^{\ell-1/2} \quad \text{for} \quad S_n < 0$$

$$X(a,b)Y : \sigma_\ell \sim k^{2\ell+1} \quad \Longleftrightarrow \quad (-S_n)^{\ell+1/2} \quad \text{for} \quad S_n > 0$$

Charge radii and neutron correlations in halo nuclei: ^{6}He and ^{8}He

G. Papadimitriou et al, PRC 84, 051304(R) (2011)

Example:

Translationally invariant Hamiltonian:

$$H = \sum_{i=1}^{A_v}\left(\frac{\mathbf{p}_i^2}{2\mu} + U_i\right) + \sum_{i<j}^{A_v}\left(V_{ij} + \frac{\mathbf{p}_i\mathbf{p}_j}{A_c}\right)$$

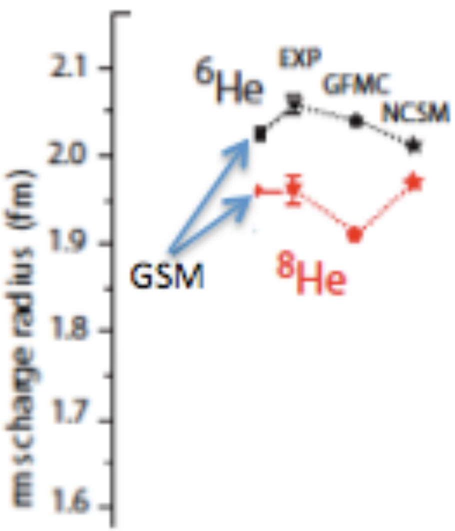
"Recoil" term

U - '^{5}He' WS potential with s.o.

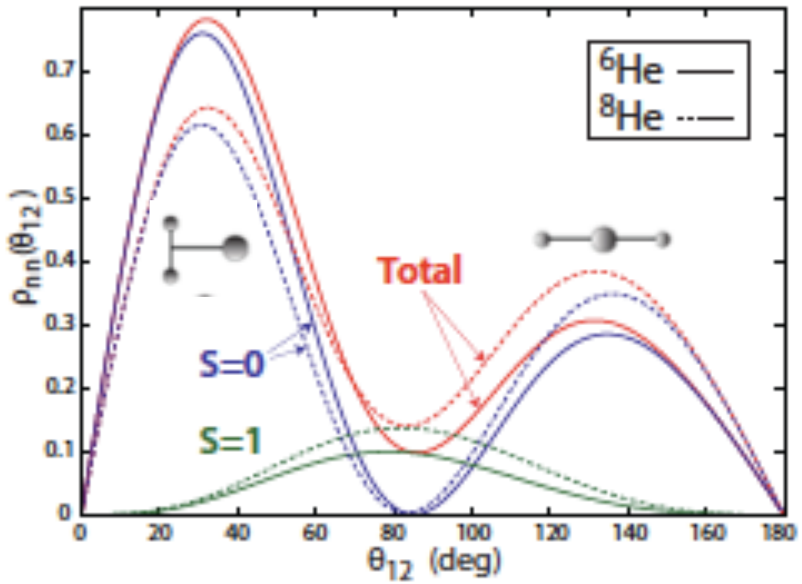
V – finite-range Minnesota int.

GSM Hamiltonian reproduces the energetics in the helium isotopic chain:

$$S_{1n}, S_{2n}, 2^+\left({}^6\mathrm{He}\right), 3/2^-\left({}^7\mathrm{He}\right), \ldots$$



$$S_{2n}\left({}^6\mathrm{He}\right) < S_{2n}\left({}^8\mathrm{He}\right)$$



Reduction of the charge radius in ^{8}He is due to a reduction of S=0 'dineutron configuration' which is strongly enhanced by the coupling to the continuum.

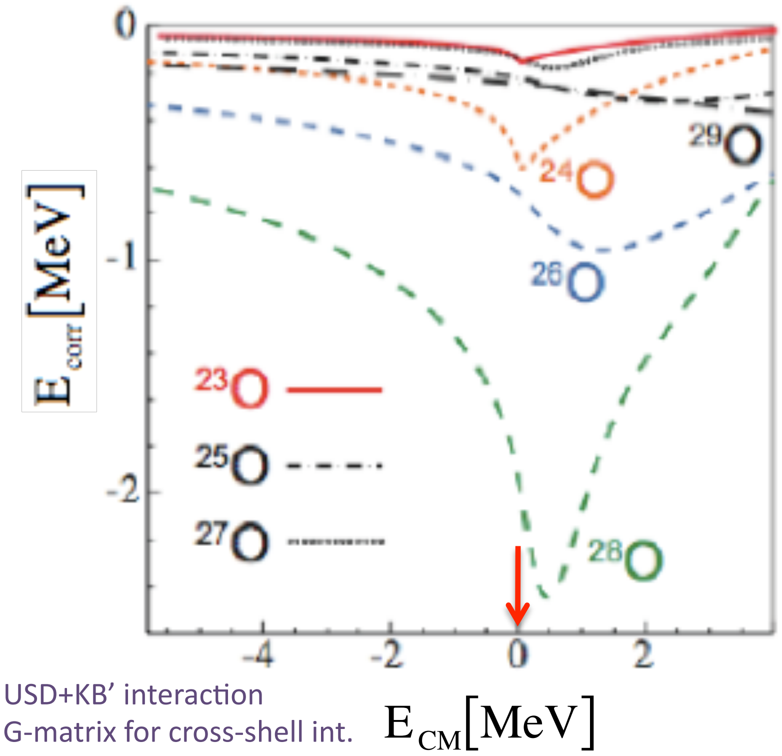
How does the continuum work?

Continuum coupling correlation energy is of the same order as the pairing correlation energy

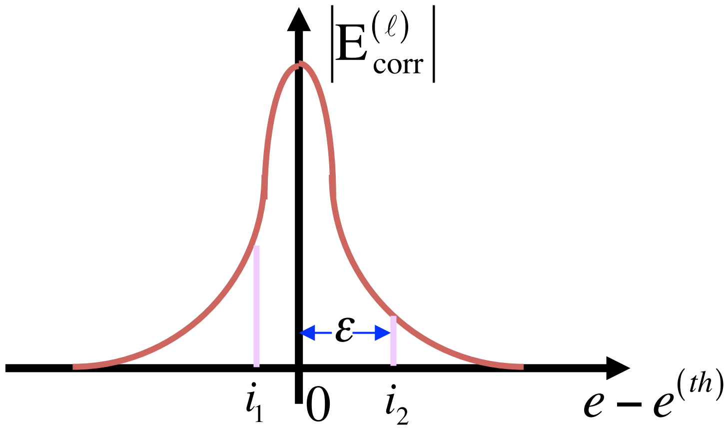
?

Instability of SM eigenstates at the channel threshold?

$$E_{corr} = \langle \Phi_i^A | \mathcal{H}_{QQ}^{eff} - H_{QQ} | \Phi_i^A \rangle$$



USD+KB' interaction
G-matrix for cross-shell int.
WB continuum coupling

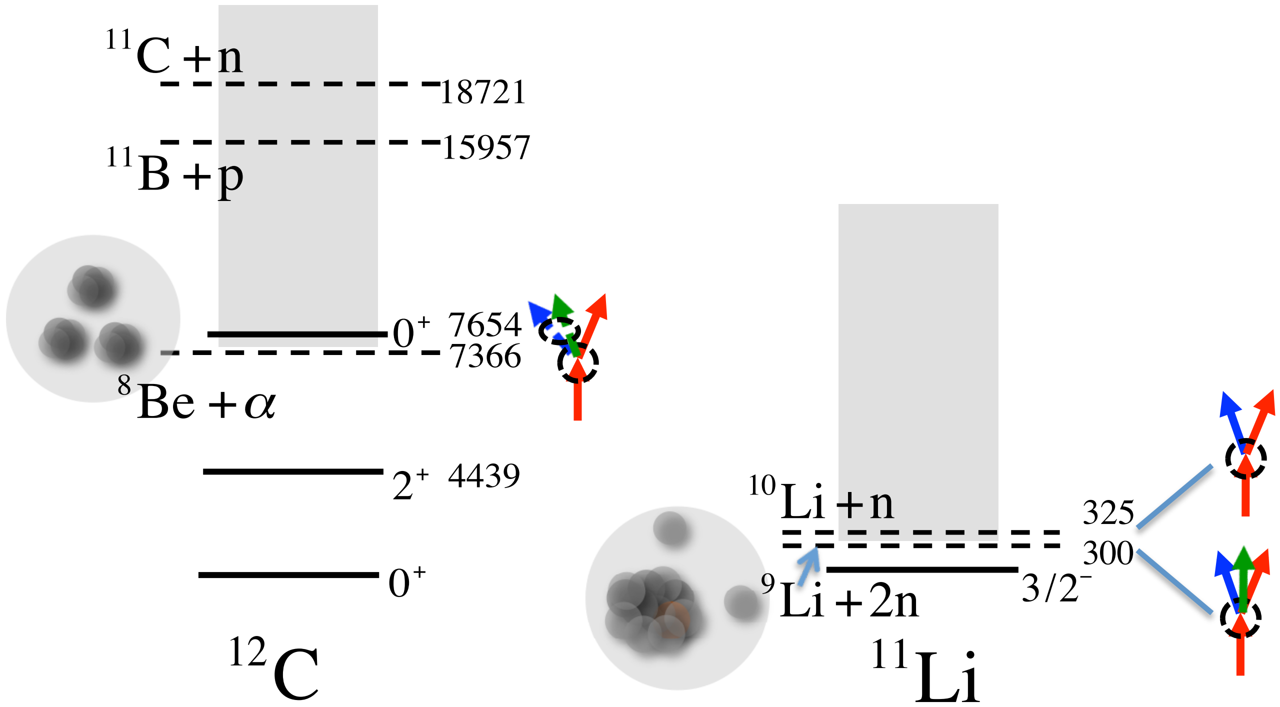


Admixture of many-body continuum states with $E > E_{th}$

Is it a collective phenomenon?

Nuclear clustering

J. Okolowicz, W. Nazarewicz, M.P., arXiv:1202.6290



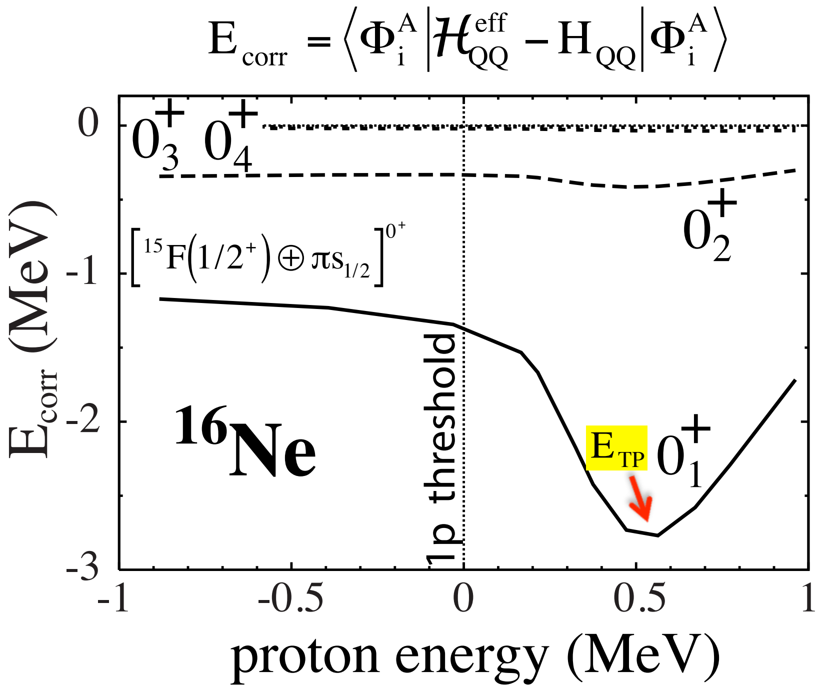
Specific:

Energetic order of emission thresholds and absence of stable cluster entirely composed of like nucleons

Generic:

Correlations in the near-threshold states depend on the nature of the nearest branching point

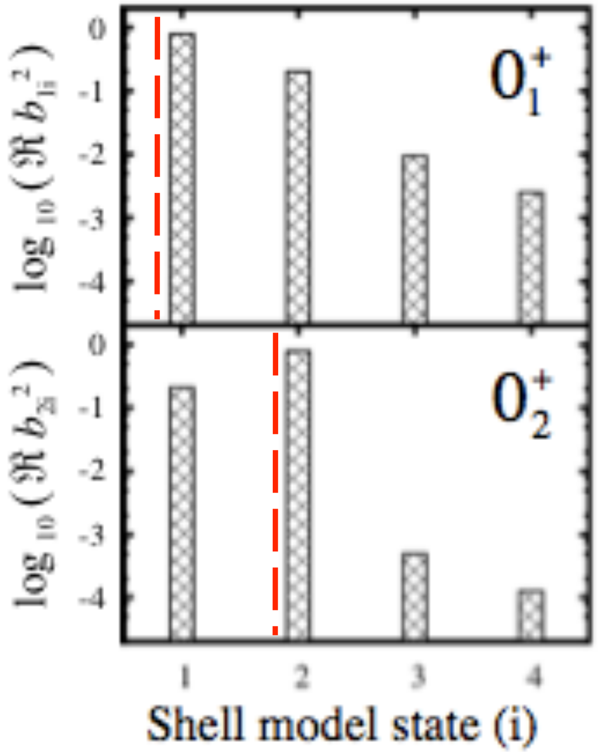
Continuum coupling correlation energy



The maximum continuum coupling point E_{TP} are determined by the interplay between the Coulomb (centrifugal) interaction and the continuum coupling

The 'aligned' state

Weights of SM states in
'aligned' CSM eigenstates



Interaction through the continuum leads to the collectivization of SM eigenstates and formation of the aligned CSM eigenstate which couples strongly to the decay channel and, hence, carries many of its characteristics.

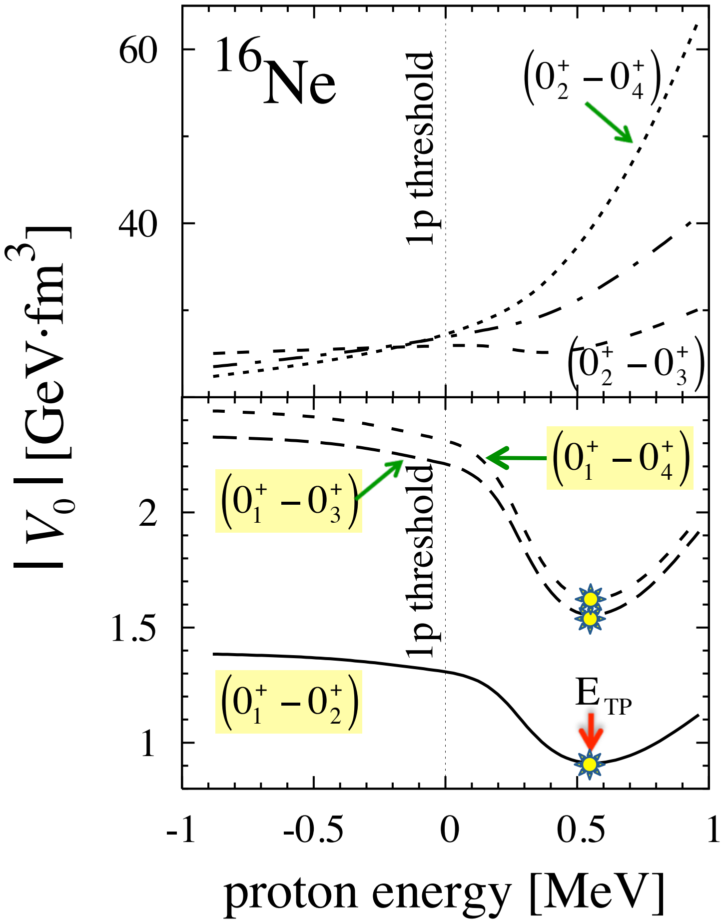
Mixing of wave functions via the continuum

- The mixing of eigenfunctions (avoided crossing) is caused by a nearby exceptional point ($\varphi_1 = \varphi_2\,(\varphi_1 = \varphi_1^*)$) of the complex-extended Hamiltonian.

- Exceptional point is a generic situation in open quantum systems.

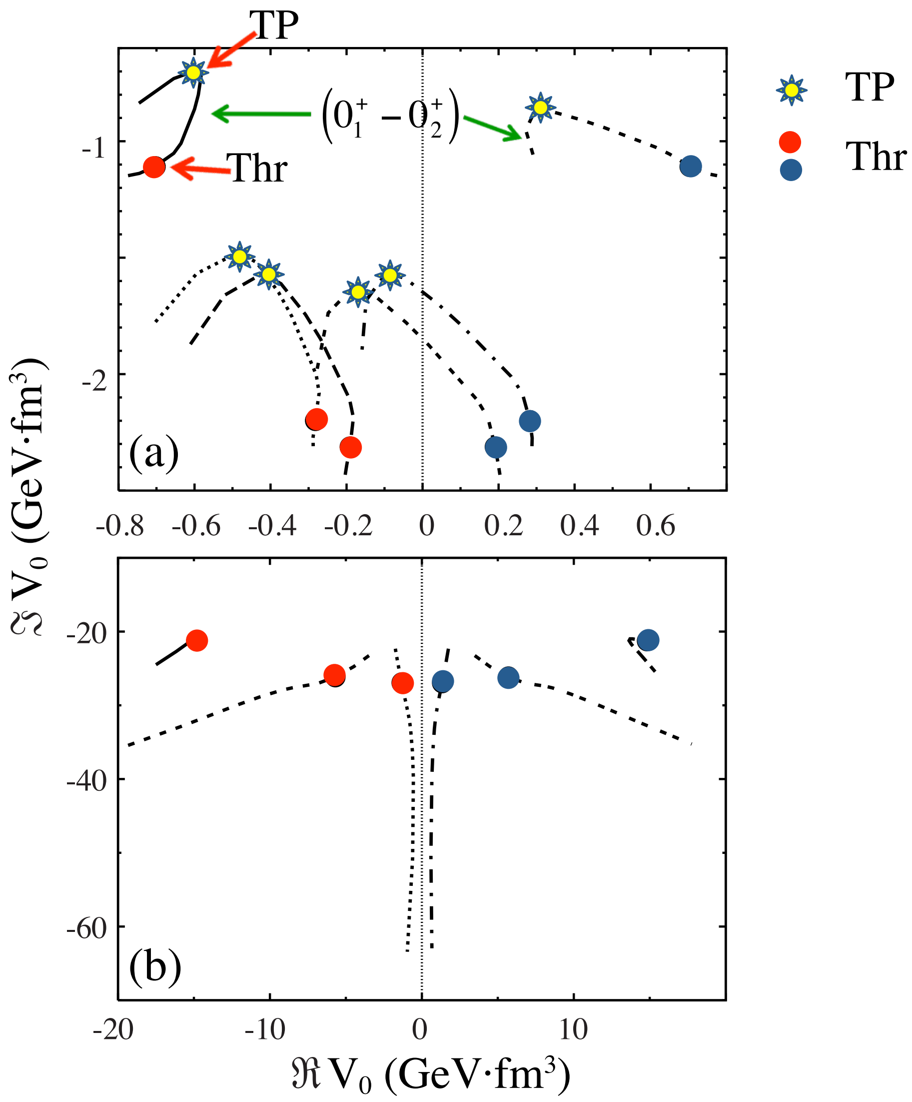
- The configuration mixing of resonances is characterized by lines $\mathcal{E}_{\alpha_1}(E) = \mathcal{E}_{\alpha_2}(E)$ of exceptional points (exceptional threads) of the complex-extended CSM Hamiltonian (complex V_0).

Mixing of wave functions via the continuum



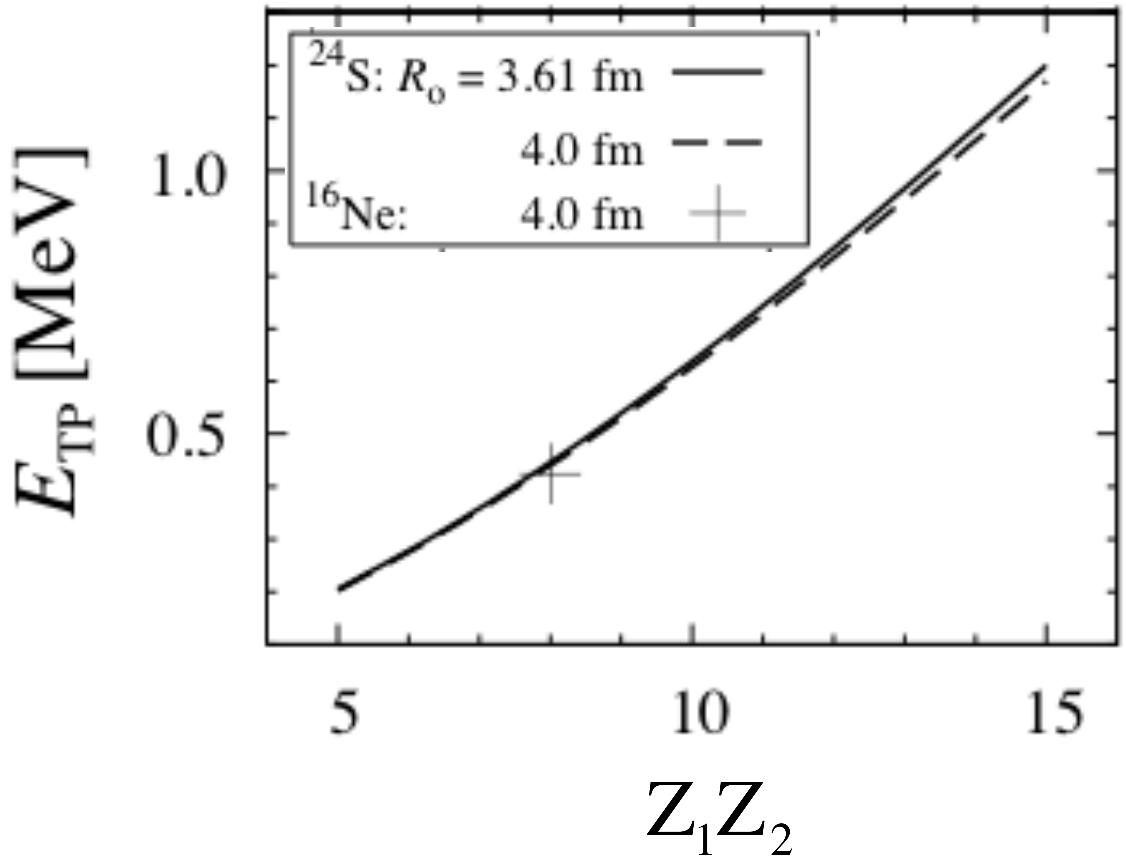
All exceptional threads which are relevant for the collective mixing of SM states involve the aligned eigenstate 0_1^+. They exhibit a minimum of $|V_0|$ at the same energy (the turning point energy).

Mixing of wave functions via the continuum



The 'window of opportunity' for charged-particle clustering is situated around the turning point above the charged-particle decay threshold.

The universality of the collective mixing of eigenstates via the continuum



For a given value of Z_1Z_2, the turning point energy depends weakly on the nature of the charged particle decay channel and the parameters of the potential.

Outlook

1. The non-resonant continuum is essential for the spectroscopy of weakly bound nuclei:
   - mixing of Shell Model states through the particle continuum, modification of the effective interaction and NN correlations
   - collective phenomena: clustering, resonance trapping, super-radiance, multichannel coupling effects in reaction cross-sections and shell occupancies, modification of spectral fluctuations
   - breaking of the isospin symmetry
   - energy shifts, coalescence of eigenvalues, …

2. The clustering is the generic near-threshold phenomenon in open quantum system which does not originate from any particular property of forces or any dynamical symmetry of the many-body problem.
   - Nuclear clustering is a consequence of the collective coupling of SM states via the decay channel which leads to the formation of the aligned eigenstate of the OQS. This state captures most of the continuum coupling and carries many characteristics of the decay channel.
   - Cluster states may appear in the narrow energy window around the point of maximum continuum coupling.

All the best George!